

Lesson 20. The Points-After-Touchdown Problem

1 The problem

- In an NFL football game, after scoring a touchdown, a team is given the option to try for:
 - a **1-point conversion**: 1 extra point by a field goal from the 15-yard line, or
 - a **2-point conversion**: 2 extra points by advancing the ball into the end zone from the 2-yard line
- Whether to “go for 2” is a classic debate – a few somewhat recent discussions on the topic:
 - <https://theringer.com/nfl-two-point-conversions-pittsburgh-steelers-mike-tomlin-65d47282d853>
 - <https://fivethirtyeight.com/features/more-nfl-teams-are-going-for-two-just-as-they-should-be/>
- Adding to the debate: in 2015, 1-point attempts were moved from the 2-yard line to the 15-yard line
- Conversion success rates from the past 4 regular seasons (from <http://www.pro-football-reference.com/>):

| | 2014 | 2015 | 2016 | 2017 |
|---------------------------------|-------|-------|-------|-------|
| 1-point conversion success rate | 0.993 | 0.942 | 0.936 | 0.940 |
| 2-point conversion success rate | 0.483 | 0.479 | 0.486 | 0.451 |

- Based on the current score and time remaining, should a team “go for 1” or “go for 2” in order to maximize the probability that it wins the game?
- How does the 2015 rule change affect a team’s optimal conversion strategy?
- Let’s try to answer these questions by modeling this problem as a stochastic dynamic program
- We will be roughly following this paper:

H. Sackrowitz (2000). Refining the point(s)-after-touchdown decision. *Chance* 13(3): 29-34.

2 Data

- Two teams: A and B
 - Assume that we (the decision-makers) are Team A
- Suppose we have the following data:

T = total number of possessions

$p_n = \Pr\{\text{1-pt. conv. successful for Team } n \mid \text{1-pt. conv. attempted by Team } n\}$ for $n = A, B$

$q_n = \Pr\{\text{2-pt. conv. successful for Team } n \mid \text{2-pt. conv. attempted by Team } n\}$ for $n = A, B$

$b_1 = \Pr\{\text{1-pt. conv. attempted by Team B}\}$

$b_2 = \Pr\{\text{2-pt. conv. attempted by Team B}\}$

$t_n = \Pr\{\text{TD by Team } n \text{ in 1 possession}\}$ for $n = A, B$

$g_n = \Pr\{\text{FG by Team } n \text{ in 1 possession}\}$ for $n = A, B$

$z_n = \Pr\{\text{no score by Team } n \text{ in 1 possession}\}$ for $n = A, B$

$r = \Pr\{\text{Team A wins in overtime}\}$

- What is the relationship between b_1 and b_2 ?

- What is the relationship between t_n , g_n and z_n ?

- What is the probability that Team B scores 0 after a touchdown?

3 The stochastic DP

- Stages:

$$\begin{aligned}
 t = 0, 1, \dots, T - 1 & \leftrightarrow \text{end of possession } t \\
 t = T & \leftrightarrow \text{end of game}
 \end{aligned}$$

- For our purposes, a possession ends when a team scores (TD or FG), or loses possession without scoring

- States:

$$\begin{aligned}
 (n, k, d) & \leftrightarrow \text{Team } n\text{'s possession just ended} && \text{for } n \in \{A, B\} \\
 & \text{Team } n \text{ just scored } k \text{ points} && \text{for } k \in \{0, 3, 6\} \\
 & \text{Team A is ahead by } d \text{ points} && \text{for } d \in \{\dots, -1, 0, 1, \dots\}
 \end{aligned}$$

- Value-to-go function:

$$\begin{aligned}
 f_t(n, k, d) = \text{maximum probability that Team A wins when in state } (n, k, d) \text{ at the end of possession } t \\
 \text{for } n \in \{A, B\}, k \in \{0, 3, 6\}, d \in \{\dots, -1, 0, 1, \dots\}
 \end{aligned}$$

- Allowable decisions x_t at stage t and state (n, k, d) :

$$\begin{aligned}
 x_t \in \{1, 2\} & \text{ if } n = A \text{ and } k = 6 \\
 x_t = \text{none} & \text{ if } n = A \text{ and } k \in \{0, 3\} \\
 x_t = \text{none} & \text{ if } n = B \text{ and } k \in \{0, 3, 6\}
 \end{aligned}$$

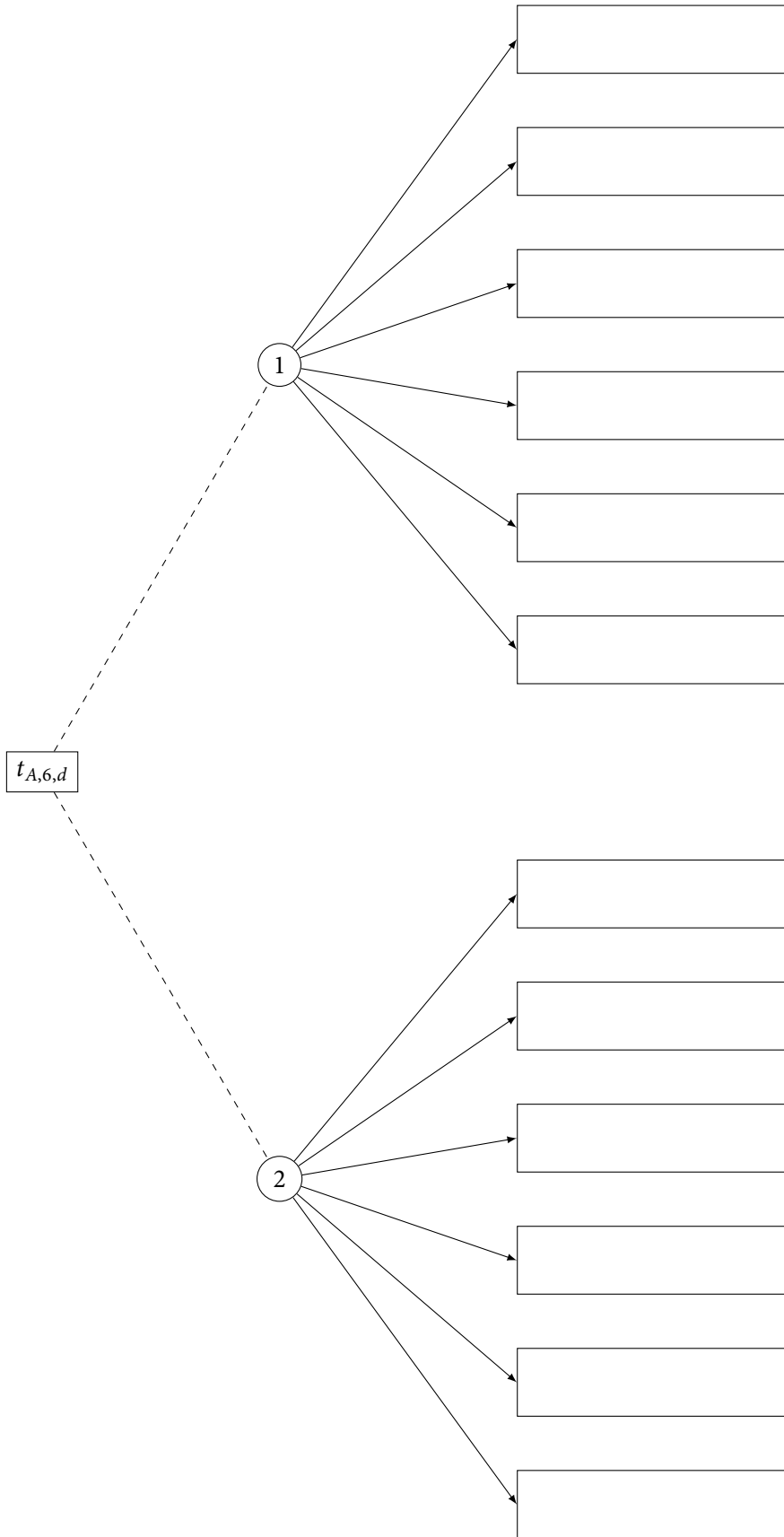
- We need to consider transitions from the following states:

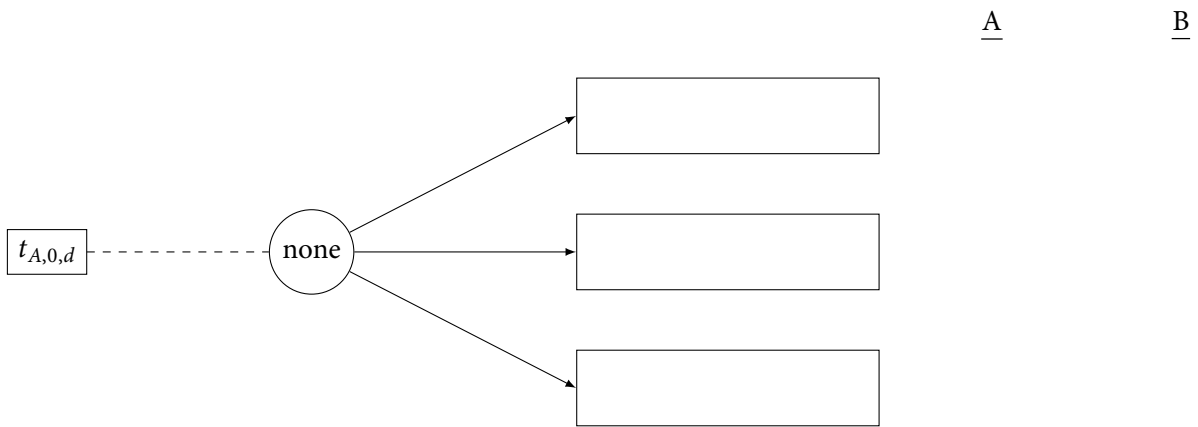
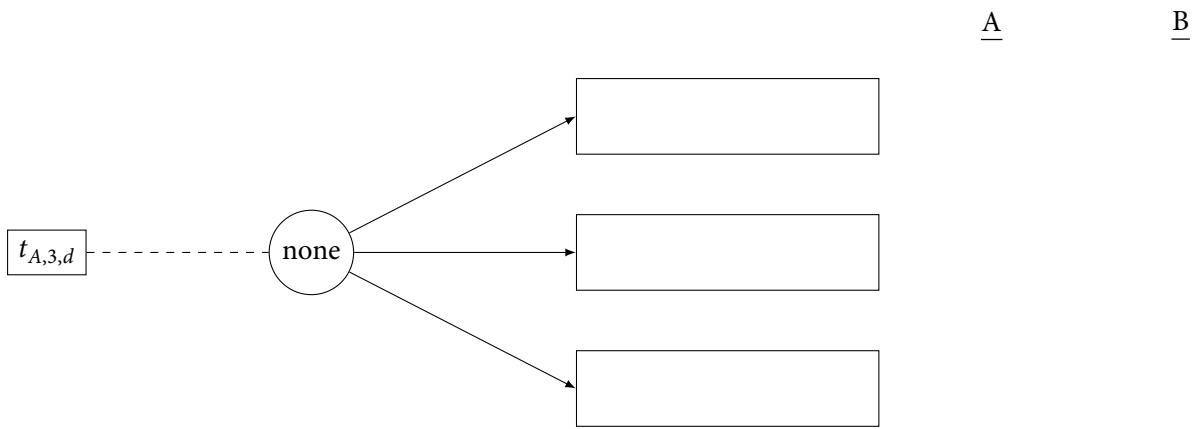
$$\begin{aligned}
 (A, 6, d) \quad (A, 3, d) \quad (A, 0, d) \\
 (B, 6, d) \quad (B, 3, d) \quad (B, 0, d)
 \end{aligned}
 \quad \text{for all } d$$

- Since our objective is to maximize the probability of winning, we set all the contributions in stages $t = 0, 1, \dots, T - 1$ to 0, just like in the investment problem in Lesson 18

A

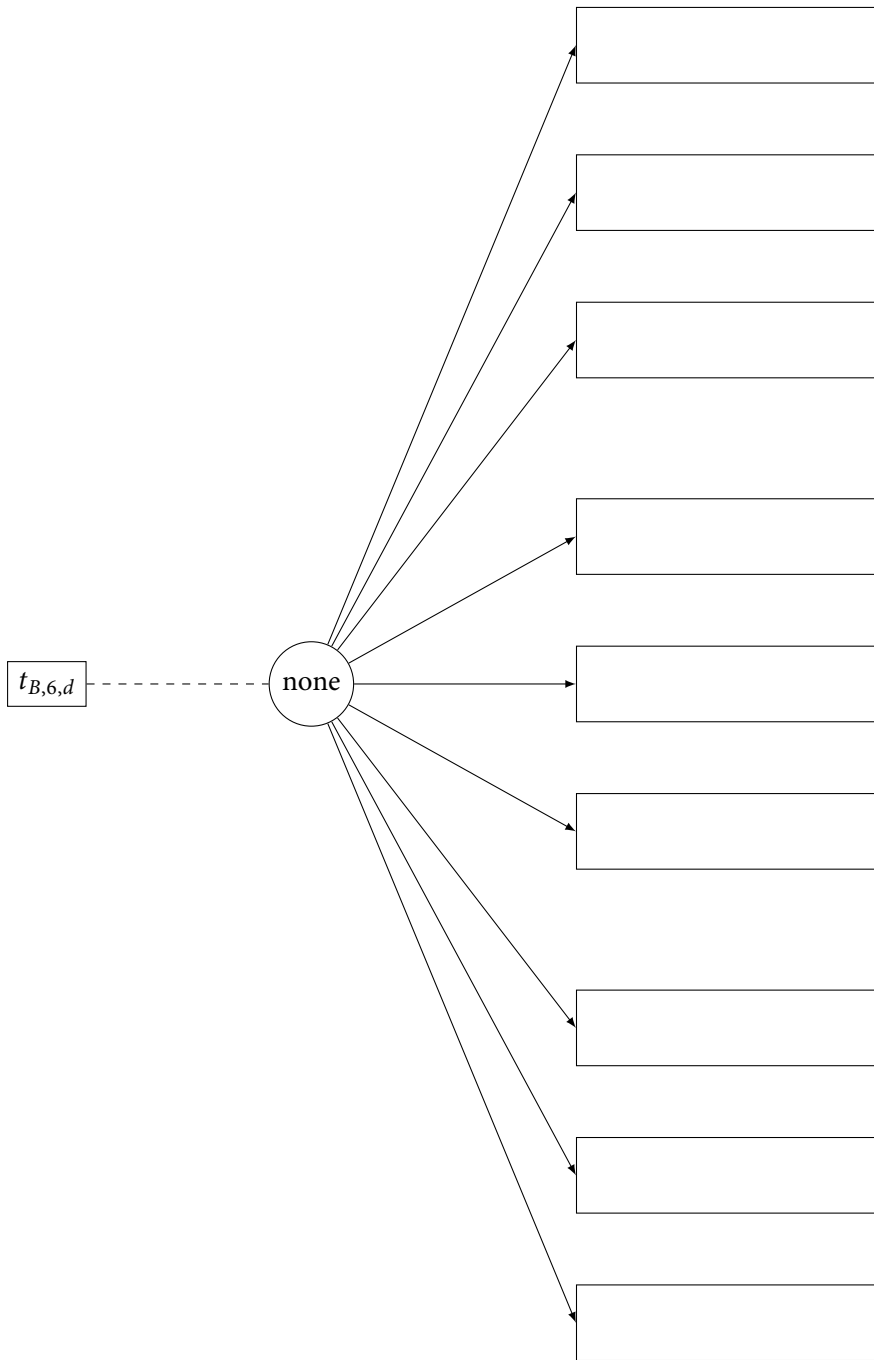
B

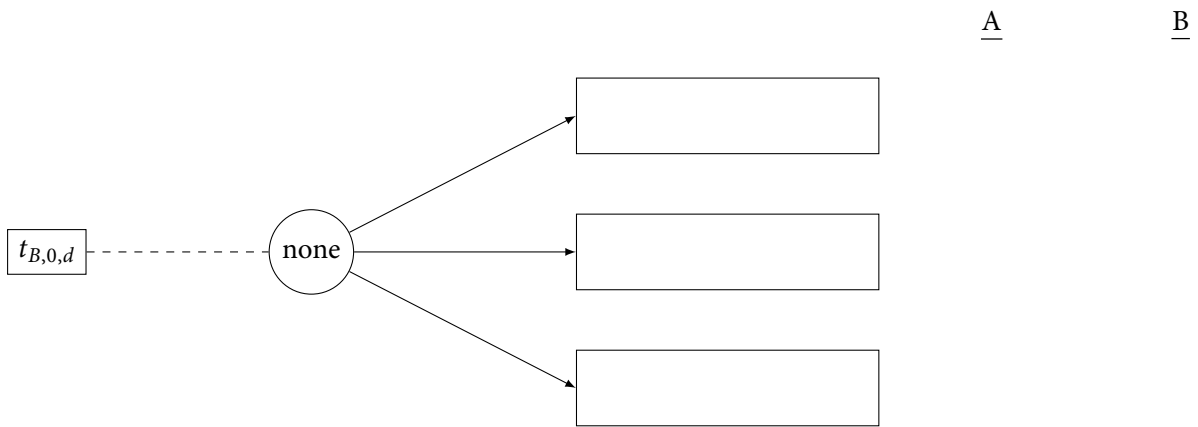
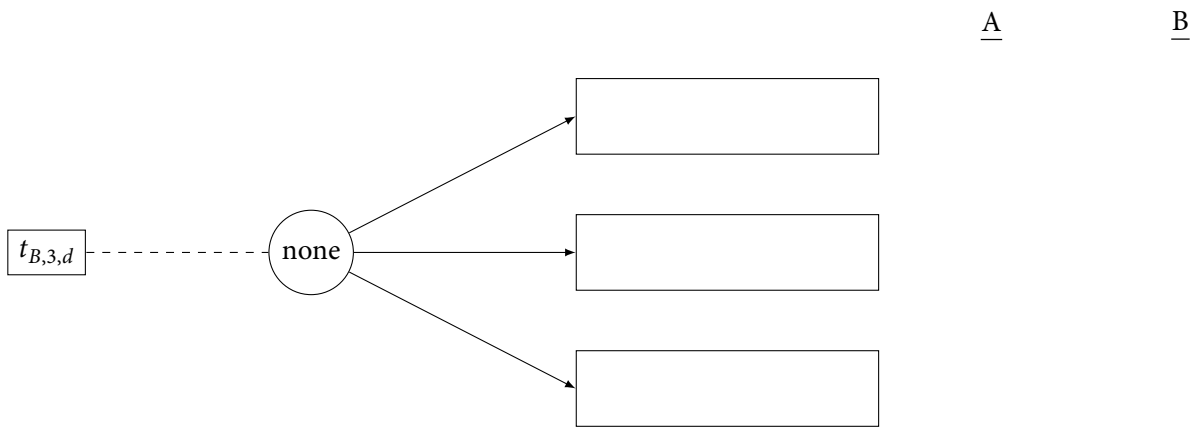


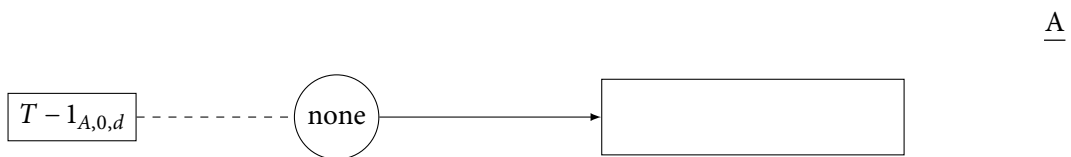
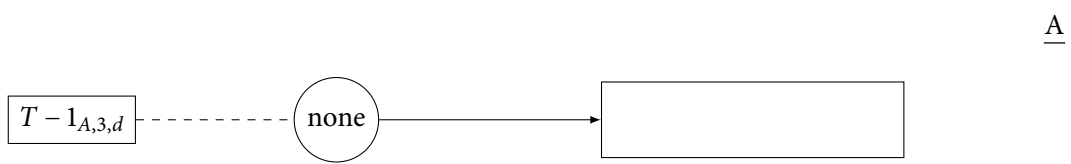
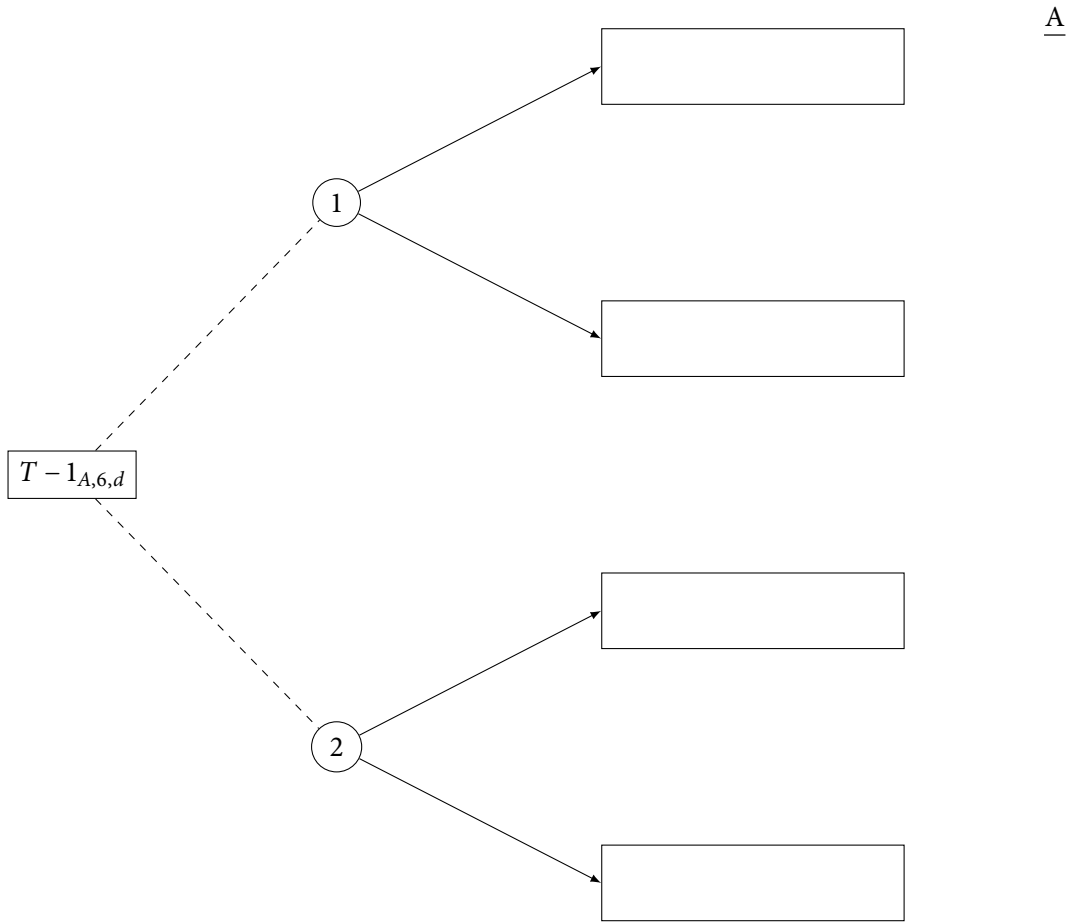


A

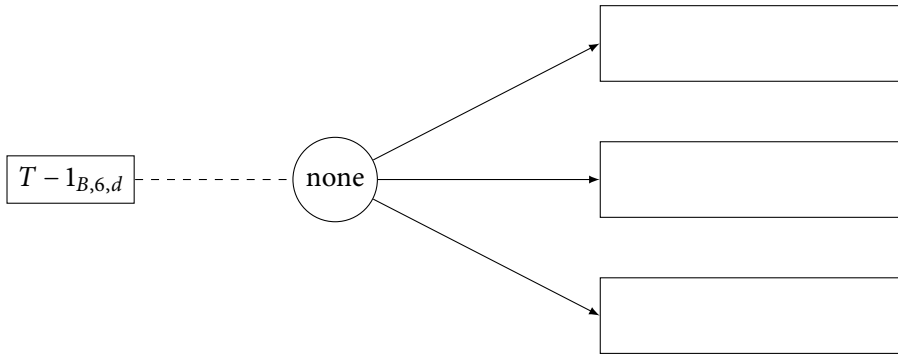
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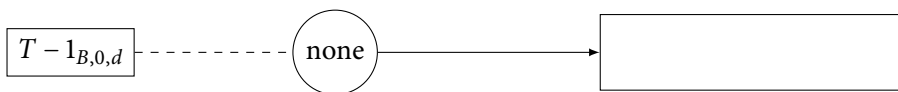
B



B



B



The outcomes of several football games each year depend crucially on extra point strategy.

Refining the Point(s)-After-Touchdown Decision

Harold Sackrowitz

In football, after scoring a touchdown a team is given the option of trying for one or two extra points. Successfully kicking the ball between the uprights from approximately 19 yards away is worth one point. Such an attempt is almost automatic at the professional level. Alternatively a team may run a regular play starting at the two-yard line. Getting the ball into the end zone on this play is worth two points. National Football League (NFL) teams were successful on 39% of two-point attempts in 1998 and 43% in 1997.

Before the 1999 NFL season was even four hours old, one game had turned on a conversion after touchdown decision. With 3:53 remaining in the third quarter, the New York Jets cut New England's lead to 27–22 and coach Bill Parcells decided to attempt a two-point conversion. He stuck with this decision even after a penalty backed them up to the seven-yard line. The attempt failed and so, when a Bryan Cox interception return for touchdown put the Jets up by one point (28–27) with 9:34 remaining in the game, they felt forced to make another two-point attempt. That attempt also failed, and the Patriots ended up winning by two points on a last-minute field goal. Had they simply



kicked two one-point conversions the game would have gone into overtime.

NFL teams collectively typically attempt more than 100 two-point conversions every year. In addition there are many other instances when the coach

at least considered (or should have considered) trying for two points. Decisions involving point-after-touchdown conversions can only affect close games — that is, games in which the two teams are tied or separated by only a couple of

points at the end of regulation time. Most games are not. Unfortunately, at a point early in the game, we cannot reliably predict the closeness of the final score. Thus all conversion decisions should be given proper attention. Relatively few games will be impacted by these decisions; however, a single unnecessary loss can devastate a football season.

To appreciate the potential impact of these decisions, one need only look at last season's ultimate game, the year 2000 Super Bowl. With the score 23–16 in favor of the St. Louis Rams and 1:54 remaining in the game, the Tennessee Titans mounted a desperation drive, which ended on the Rams' one-yard line as time ran out. Had they scored a touchdown and added the extra point, the game would have gone into overtime. Less exciting but of greater interest here is the two-point conversion attempt by Tennessee when they were trailing 16–6 with 14 seconds still remaining in the third quarter. In retrospect we see that if they had instead simply kicked an extra point, as suggested by the analysis performed here, then that last yard they were fighting for would likely have given them a win rather than just a tie.

It is easy to criticize decisions after the fact. This article describes and illustrates an approach for developing reliable information to improve coaches' decision making. The following sections describe the current approach and the issues involved. Then an approach based on dynamic programming is described.

The Current Approach and Limitations

Most NFL teams appear to rely on a combination of simplistic tables and intuition to guide their two-point conversion decisions. Both of these tools are flawed. The tables that most coaches have available for advice depend only on the current score. A decision that is correct for a given score late in the game need not be correct early in the game with the same score. As for intuition, it is often based on a coach's experiences at the end of games. Late in a game there is not much that can happen in the remaining time. Thus if a decision is

needed late in a game it is not usually difficult to determine the correct action. The more time remaining, the more difficult the analysis. Unfortunately specific situations with much time remaining do not arise often enough for even the most experienced football people to have developed (empirically) reliable intuition.

Probabilistically, point-after-touchdown conversion decisions range from the obvious to the extremely delicate. Any optimal strategy must depend, not only, on (1) the current point differential but also on (2) the number of possessions remaining in the game and (3)

Most NFL teams appear to rely on a combination of simplistic tables and intuition to guide their two-point conversion decisions. Both of these tools are flawed.

the probability distribution of all possible occurrences during that time. When all of these factors are taken into consideration, some commonly made decisions become questionable. We will see how the method of dynamic programming can be used to develop tables that should be very helpful to coaches making these decisions.

The Range of Decisions

Before beginning the analysis we look at some situations that even the casual fan should recognize. Let us start with the simplest case. In a Jacksonville Jaguar–Carolina Panther game during the 1999 season, the Panthers scored a touchdown with 31 seconds on the clock to bring the score to 22–20 in favor of the Jaguars. It did not take a rocket scientist to realize that Carolina had to attempt a two-point conversion. Of course, had the score been 21–20 at that

point, we still wouldn't need the rocket scientist, but we would have to think a bit. Is the probability of a successful two-point conversion attempt greater or less than the probability of winning in overtime?

Bobby Ross, of the Detroit Lions, was faced with a nearly equivalent dilemma in the Lions game against the Arizona Cardinals (11/14/99). With 5:26 left in the game, the Lions scored a TD to pull within 23–19 of Arizona. They went for two and missed. As a result, the Lions lost 23–19 because they could not go for a field goal (worth three points) when they got to the Cardinal 10-yard line with under two minutes remaining. Ross had been hoping for one more possession and for an outright win. Due to injuries, he said he did not want to play in overtime. In effect he determined that his team's probability of a two-point conversion was greater than his team's probability of winning in overtime. Despite being severely criticized, he may have been correct about that decision. Certainly he did consider the appropriate factors. To find the truly questionable decision (which went unnoticed), we have to go back to the third quarter. With 10:51 remaining in the third quarter, the Lions scored a TD to cut the Arizona lead to 23–13. With a 10-point deficit, naïve tables would tell you to go for a two-point conversion. This would be the proper action near the end of a game. It is not, however, the percentage play early in the second half. Had they kicked an extra point after that TD, they also would have simply kicked again in the fourth quarter, and they likely would have won the game with a last-minute chip-shot field goal.

A 12-point lead seems to provide a great temptation to attempt a two-point conversion at any time of the game. The wild-card playoff game in 1998 between the Jacksonville Jaguars and New England Patriots provides a good illustration. When ahead 12–0 with 5:58 still remaining in the second quarter the Jaguars attempted a two-point conversion that failed. When ahead by 12 points *late* in a game it is appropriate to try for two points. Calculations show that decision to be questionable, however, when made early in the game. Although Jacksonville went on to win, I'm sure they realized the needless danger they put themselves in when, with

almost a full quarter remaining, New England pulled to 12–10 and seemed to have the momentum. In 1997 Oakland actually lost a game to the Jets partly because they made the same decision very early in the game.

The Goal

The examples described previously provide ample anecdotal evidence that better decision-making tools are required. The goal here is to develop a method that will rely less on intuition and improve a coach's decision-making ability. It is not really possible to find exact optimal strategies because the needed parameters (e.g., scoring probabilities) are unknown and must be estimated. Under the best of conditions, these parameters are difficult to estimate empirically, particularly early in a season. Furthermore, they may vary during the season (e.g., due to player injuries). The coach's knowledge and intuition will still have to be used. The model and approach to be described will result in tables that can give a coach a far more realistic sense of the risks involved at each point in the game.

The most perplexing issue of the two-point conversion decision for a coach is how to factor in time remaining. Near the very end of a game the correct choice is usually clear. As far as time remaining, there seem to be two options. The time can be measured in real time (i.e., minutes and seconds) or in possessions (which must be estimated). If we choose real time (which would be known exactly), the analysis would then require estimating a probability distribution for the length of a possession. I have opted to measure time in possessions.

Model and Methods

The method of dynamic programming (or backward induction) is particularly well suited for deriving optimal strategies in games with time limits. It has been used effectively, for example, by the author in a *Chance* article (Vol. 9, No. 1, Winter 1996) to analyze time management in sports. The basic idea is to determine optimal strategy, first for the last possession of the game, then for the previous possession, and so forth.

The key decision points are after a team has scored a touchdown. It turns out, however, that our method requires thinking about each time the ball changes possession. At such points the game situation can be described as (k, d, t) , where k is the number of points scored immediately prior to the change of possession and prior to any point after touchdown attempt ($k = 0$ for no score, $k = 3$ for a field goal, $k = 6$ for a touchdown), d is the lead of Team A, and there are t possessions remaining in the game. The objective is to win the game; let $V(k, d, t)$ be the probability that Team A wins given it is giving up possession of the ball in situation k, d, t and both teams make optimal decisions from this point on. Similarly $V^*(k, d, t)$ is the probability that Team A wins given that Team B is finishing its possession with situation (k, d, t) and both teams make correct decisions from here on out. Team A wants to make $V(k, d, t)$ as large as possible and will make conversion decisions accordingly; Team B wants to make $V^*(k, d, t)$ as small as possible.

We provide a brief description of the dynamic programming approach here. More details are provided in the sidebar. Suppose we have determined the optimal strategies for both teams and the resulting probabilities V and V^* with $t - 1$ (or fewer) possessions remaining in the game. Now suppose Team A scores a touchdown ($k = 6$) to make the lead -10 points ($d = -10$) with one quarter (approximately $t = 6$

possessions) remaining. Team A in this case could be Tennessee in the 2000 Super Bowl. The situation with $t - 1$ or 5 possessions remaining will depend on three things: (1) the conversion strategy selected by Team A (one-point or two-point), (2) success or failure of the chosen strategy, and (3) the result of Team B's next possession. Suppose Team A attempts a one-point conversion. Then it will be successful (with probability .98) or not (with probability .02). Following that, Team B will fail to score (with probability .68), score a field goal (three points with probability .12) or score a touchdown (six points with probability .20). To demonstrate the key step in dynamic programming, consider what happens if the conversion is successful and Team



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Table 1 — Optimal Conversion Strategy for an “Average” 1998 Team.

Entries give conversion (1 or 2 pt) to be attempted. Blank indicates it doesn't matter.

| status after scoring TD | total number of possessions remaining in the game for both teams combined | | | | | | | | | | | | | | | | | | |
|-------------------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|
| behind by | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 15 | | | | | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 14 | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 13 | | | | | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 12 | | | | | | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| 11 | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | | | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | | | | | 2 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| 4 | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| ahead by | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | | 1 | 1 | 1 | 2 | 1 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 |
| 6 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 11 | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 12 | | | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |

Note 1: The status is immediately after the TD but before any conversion is attempted.

Note 2: To improve intuition we point out that a typical NFL game has approximately 6 possessions in each quarter.

B scores a touchdown. This happens with probability (.98 x .20 = .196), and the probability that A wins after these two events is $V^*(6, d - 5, t - 1)$; this is the probability that A wins when B is to give up possession after a touchdown with A's lead $d + 1 - 6 = d - 5$ and $t - 1$ possessions remaining. But remember we assumed that we already know all values of V and V^* with $t - 1$ possessions remaining. Then A should evaluate its strategies by computing the expected probability of winning for each strategy. This just requires finding the probability of each possible out-

come for the strategy (conversion successful or failed, Team B scoring 0, 3, 6 points) and the probability of winning after each possible outcome. See the sidebar for additional details.

Results

Table 1 is an example of what a time-dependent strategy table would look like. It was constructed for two hypothetical teams having the scoring characteristics of two “NFL league average” 1998 teams playing against one another.

Each team averages 12 possessions per game. They each score on average 2.41 touchdowns per game (the probability of a touchdown on a possession is $2.41/12 \approx .20$), make 1.475 field goals per game (the probability of a field goal on a possession is $1.475/12 \approx .12$), and convert 98% of their one-point conversion attempts and 39% of their two-point attempts. Similar tables can be constructed for particular matchups involving specified teams by applying dynamic programming methods with different scoring rates and conversion success rates. One difficulty, as mentioned ear-

Dynamic Programming for Finding Optimal Point-After-Touchdown Strategies

The accompanying article uses dynamic programming to determine optimal point-after-touchdown conversion strategies. The approach is described in more detail here. The first step is to carefully define the set of possible situations or states. We focus on the moments during the game when the ball will change possession. Specifically, consider the moments immediately after either team scores a field goal (FG, worth three points), immediately after either team scores a touchdown (TD, worth six points) but before any extra points have been attempted, and immediately after a possession change with no scoring (NS). At any of these moments we will say we are in situation (k, d, t) meaning that we have just scored k points ($k = 0, 3, \text{ or } 6$), there are t possessions remaining in the game and Team A is ahead by d points. A negative d means that Team A is behind. We will talk about having to make decisions in situation (k, d, t) even though a choice of options is available only after a TD (i.e., $k = 6$). Note that this does ignore some rare events (e.g., a safety).

Let $V(k, d, t)$ be the probability that Team A wins when Team A is to turn the ball over in situation (k, d, t) and both teams behave optimally in this and all future situations (remember that Team A has real options only if it has scored a TD to reach this situation). Similarly, let $V^*(k, d, t)$ be the probability that Team A wins when Team B is to turn the ball over in situation (k, d, t) and both teams behave optimally in this and all future situations. Team A wants $V(k, d, t)$ to be large while Team B wants $V^*(k, d, t)$ to be small. Say that Team A has just scored a TD and is in situation $(6, d, t)$. At this point Team A will attempt either a one- or two-point conversion and add 0, 1, or 2 points to its score. Team B will then get the ball and during the next possession score a TD, a FG, or nothing. Thus adding 6, 3, or 0 points, respectively, before the next decision situation. This next decision opportunity would, of course, belong to Team B. The table below summarizes the possible situations with $t - 1$ possessions remaining.

| Result of extra point attempt by Team A in situation $(6, d, t)$ | Result for Team B during next possession | Probability that Team A wins at the next decision situation |
|--|--|---|
| Attempt fails | No score | $V^*(0, d, t - 1)$ |
| | FG | $V^*(3, d - 3, t - 1)$ |
| | TD | $V^*(6, d - 6, t - 1)$ |
| Makes 1-point attempt | No score | $V^*(0, d + 1, t - 1)$ |
| | FG | $V^*(3, d - 2, t - 1)$ |
| | TD | $V^*(6, d - 5, t - 1)$ |
| Makes 2-point attempt | No score | $V^*(0, d + 2, t - 1)$ |
| | FG | $V^*(3, d - 1, t - 1)$ |
| | TD | $V^*(6, d - 4, t - 1)$ |

The advantage of dynamic programming is that the process begins at the end of the game. When there are zero possessions left in the game, the optimal strategies for both teams and the probability of winning are clear. Next we find the optimal strategies and probability of winning when one possession remains, then when two possessions remain, and so forth. Once the values of $V(k, d, t - 1)$ and $V^*(k, d, t - 1)$ are determined and the optimal strategy is specified for all k and d for $t - 1$ possessions remaining, then the optimal strategies are determined when there are t possessions remaining. I shall use situation $(6, d, t)$ to demonstrate the process. Let p_1 and p_2 denote the probabilities, for Team A, of successful one-point and two-point conversion attempts, respectively. Using the table above, it is easy to see that the probability Team A wins for a given choice of conversion strategy depends only on values of V^* with $t - 1$ possessions remaining (as well as p_1, p_2 , and the probability of B's various outcomes). The key point is that values with $t - 1$ possessions remaining are known quantities.

Thus we compute the two probabilities:

- (1) $\Pr(\text{Team A wins if it tries a one-point conversion at } (6, d, t))$
 $= p_1[\Pr(\text{TD by B})V^*(6, d - 5, t - 1) + \Pr(\text{FG by B})V^*(3, d - 2, t - 1) + \Pr(\text{NS by B})V^*(0, d + 1, t - 1)]$
 $+ (1 - p_1)[\Pr(\text{TD by B})V^*(6, d - 6, t - 1) + \Pr(\text{FG by B})V^*(3, d - 3, t - 1) + \Pr(\text{NS by B})V^*(0, d, t - 1)]$
- (2) $\Pr(\text{Team A wins if it tries a two-point conversion at } (6, d, t))$
 $= p_2[\Pr(\text{TD by B})V^*(6, d - 4, t - 1) + \Pr(\text{FG by B})V^*(3, d - 1, t - 1) + \Pr(\text{NS by B})V^*(0, d + 2, t - 1)]$
 $+ (1 - p_2)[\Pr(\text{TD by B})V^*(6, d - 6, t - 1) + \Pr(\text{FG by B})V^*(3, d - 3, t - 1) + \Pr(\text{NS by B})V^*(0, d, t - 1)]$

If the probability in (2) is greater than that in (1), then attempting a two-point conversion is optimal for Team A in situation $(6, d, t)$. If the probability in (1) is greater than that in (2) then attempting a one-point conversion is optimal for Team A in situation $(6, d, t)$. If they are equal it doesn't matter.

A similar analysis will lead to Team B's optimal strategy for each k, d at time t . Then we back up to find the optimal strategies for all situations with $t + 1$ possessions remaining, and so forth.

lier, is that scoring characteristics cannot be known exactly. One way around this problem would be to prepare several tables using a range of scoring characteristics for any game. This should give a coach a much better sense of the appropriateness of conversion decisions than is now available.

Table 1 shows that, in the example that started this article, Parcels's initial decision to go for two points near the end of the third quarter was correct. Once the Jets committed the penalty that backed them up an additional five yards on the conversion attempt, however, the probabilities changed. Our table no longer applies. Although penalties (for or against the kicking team) on the extra point play are rare they can dramatically affect the probabilities and, hence, the correct decision. With a little more effort the preceding methods can also be used to develop tables to recommend strategy when a penalty has occurred.

Incidentally, tables would look much different for college and high school football. Their kicking is much less "sure," and two-point conversions are favored more often.

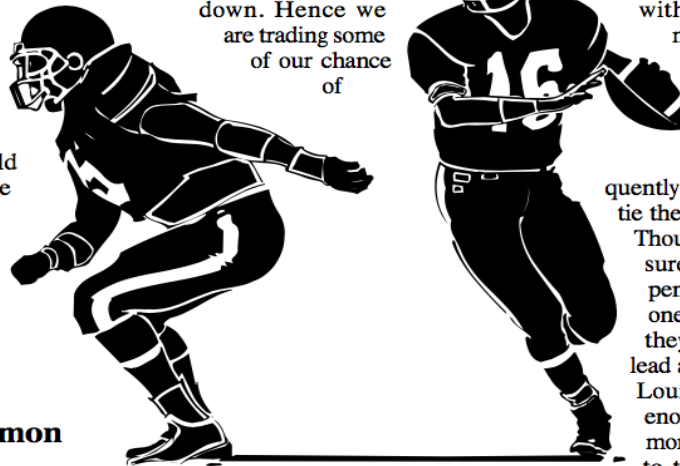
An Overlooked Strategy for a Common Situation

An interesting finding relates to a very familiar NFL occurrence. It suggests the increased use of a strategy rarely seen in the NFL. We get possession of the ball at some point during the fourth quarter, and we are behind by 14 points. Other than the obvious need to score a bunch of points, what should we be anticipating in terms of strategy? If this were a bridge column, the analyst would say that we have a fairly hopeless hand but that we should envision those distributions of cards that would permit us to make our contract. We should then play our cards accordingly. We employ an analogous approach. First assume that we will be able to outscore our opponent by two touchdowns. Thus the only real strategy issue is what to do, in

terms of extra point attempts, after each TD. How do we resolve this question?

For simplicity let's assume that a one-point conversion attempt is successful 100% of the time and that, if we make it that far, the probability of winning in overtime is $1/2$. Thus if all goes well (i.e., we score our two TD's and our opponent does not score) and we go for one-point conversions our probability of winning the game is equal to $1/2$. This seems to be standard procedure in the NFL.

Let me propose a different plan. Go for two points after the first touchdown. If successful, a one-point conversion following the second touchdown will win the game outright. If the first two-point conversion attempt fails, we still can reach overtime with a successful two-point attempt after the second touchdown. Hence we are trading some of our chance of



strategy described here. This could have been useful information, for example, for the 1998 Carolina Panthers who often found themselves 14 points behind in the fourth quarter (three times — once against Dallas and twice against Green Bay). They always went for one point after scoring a touchdown. In one of the games they actually made it to overtime but lost. In 1999 the Eagles suffered a similar fate in Washington. Reaching overtime may be a full moral victory, but it is worth only half a real victory.

Conclusion

To close we return to the 2000 Super Bowl when Tennessee scored to cut St. Louis's lead to 10 points with one quarter (approximately six possessions) remaining. Table 1 recommends a one-point conversion. Tennessee tried for two points and failed. They subsequently scored 10 more points to tie the game but ultimately lost. Though we can never know for sure what would have happened if Tennessee tried for one point, it seems likely that they might have earned the lead at 17–16. What would St. Louis have done then? It is enough to convince me that more attention should be paid to the conversion decision, perhaps using the dynamic programming approach laid out here.

reaching overtime for some chance of an outright win. In the days before overtime was an option in professional football, this strategy was studied by Porter (1967) in the case of exactly four possessions remaining. In that case it turns out that if the probability of a successful two-point conversion is more than .382 then our probability of winning is greater than $1/2$. The average success rate for two-point conversions in the NFL was 39% in 1998 and 43% the year before.

Our results in Table 1 suggest a one-point conversion after the first touchdown (cutting a 14-point deficit to 8 points) until the last one-third of the game at which point the recommendation changes to the two-point conversion

References and Further Reading

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